Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

GRAVITATIONAL ATTRACTION IN CONNECTION WITH THE RECTANGULAR INTERFEROMETER

By Carl Barus

Department of Physics, Brown University

Communicated September 14, 1918

1. Introduction.—The ease with which the rectangular interferometer admits of the measurement of small angles, induced me to adapt an apparatus with reference to it, for the measurement of the Newtonian constant. In addition to the usual system suspended in air, I also tested a floating system. Accurate work is scarcely to be expected in this laboratory where temperature variations and the agitation of the room conflict with the condition of its obtainment. But the trial is nevertheless interesting and the final work may be done elsewhere.

2. Apparatus.—There is room here only for the well known quartz fiber method of Boys, using a light needle, weighing when loaded but 1.9 grams. This was made of a rigid shaft of straw *mm*, figure 1, 26 cm. long, the ends being split slightly into four symmetrical segments each, which receive the two shots *m* and *m'* additionally secured with a little wax. The two light mirrors *b* and *c* were differently mounted: *b* on a fine pin *d*, snugly fitting corresponding perforations in the straw, was thus capable of rotation around the vertical axis (*d*) and moving up and down slightly. The mirror *c*, however, was mounted on a thin elastic strip of aluminum, clasping the shaft as shown in section at *e'*, *c'*. This is thus capable not only of rotating on a horizontal axis, but of being placed at different distances (moving right and left) from *d* to accommodate the rays of the interferometer, to which *b* and *c* are to be normal.

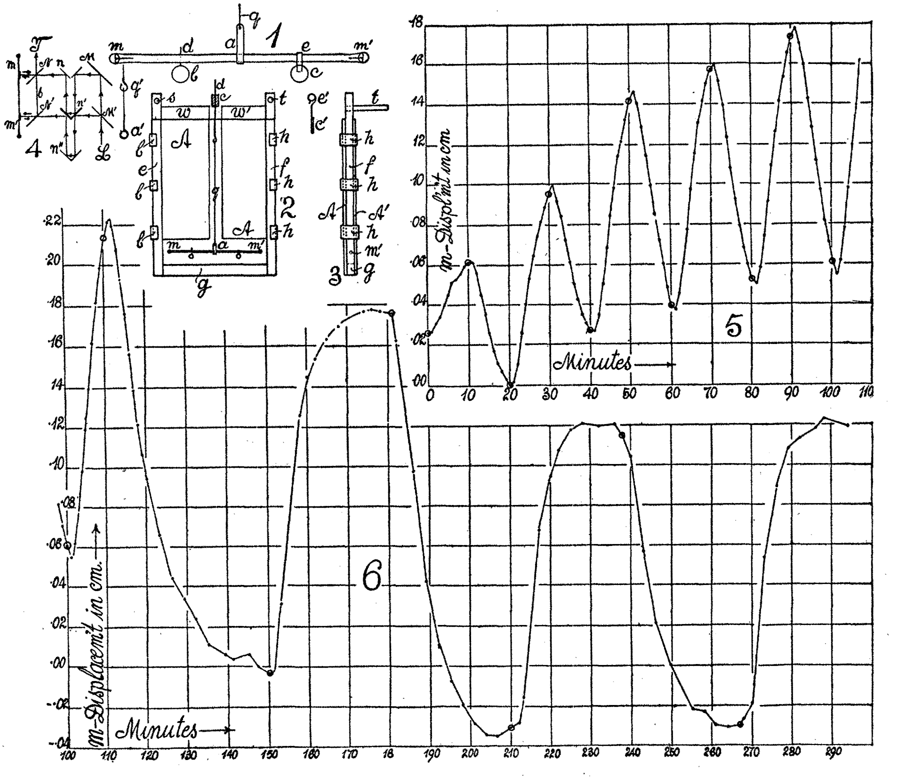
The needle is swung from a quartz fiber *q*, and a strip or hanger of elastic aluminum, *a* (see *a'*, *q'*), which clasps the shaft. Hence the latter may also be moved endwise to be balanced and rotated around a horizontal axis, till *b* and *c* meet the rays normally.

These operations are completed by trial before the needle is definitely hung, preferably in a broad beam of sunlight. No great accuracy is required.

The shallow case is made of two plates *AA*, figures 2, **3** (side view), of plate glass about 32 cm. broad and 45 cm. high, spaced by two strips *e*, *f*, 3 cm. wide of wood 1.2 cm. thick, reaching from top to bottom. Six steel clips, *b*, *b*, *h*, *h*, (such as are used for binding pamphlets), held these strips in place. At the projecting tops of the wooden strips, two nipples *s* and *t* of $\frac{1}{4}$ inch gas pipe fixed normally to the strips, served for the hanging of the case and needle from a firm wall bracket. The bottom of the case is subsequently to be closed from below by a strip of felting *gg*. To diminish the space within the case, two thin cloth covered wooden boards *w*, *w*, are inserted from the top.

The quartz fiber q to carry the needle mm, hangs from a long one-eighth brass rod d, which may be raised or lowered in view of the sleeve c held by a separate adjustable arm, without. The rod d must fit the perforation in the cork nicely, so that the former may be smoothly raised or lowered and held in any position in virtue of friction.

To swing the needle this is first placed on cork Y's, below the opening of the case AA, the felting gg having been removed. The quartz fiber q is then lowered on the long stem d, until the lower hook on the fiber is in position to grasp the clasp on the needle mm still below the case. The needle is then cautiously lifted by raising d. After this the felt strip gg is inserted to close the case, and the necessary adjustment made at c and d to swing the needle freely in the restricted space provided for it.



In the rectangular interferometer used (fig. 4) the breadth of the ray parallelogram was $b = 10$ cm. and the angle of incidence $i = 45$ degrees, so that if the micrometer displacement ΔN corresponds to the small twist θ of the needle, $\theta = 0.071\ \Delta N$.

3. *Observations.*—The quartz fiber used (diameter $2r$) was $L = 17$ cm. long. Supposing the distance apart of attracting weights M and attracted weights

m, to be $d = 6$ cm., and $M = 10^3$ grams, if the rigidity of quartz is taken as 5×10^{11}, $\gamma = 6 \times 10^6 r^4 \Delta N$. Thus if

$r = 10^{-2}$ cm., $\Delta N = 1.1 \times 10^{-6}$ cm. per kg. of M, Period $T = 4.6$ sec.

$r = 10^{-3}$ cm., $\Delta N = 1.1 \times 10^{-2}$ cm. per kg. of M, $T = 465$ sec.

The first case would then correspond to but 1/40 of a fringe; in the second case there should be 250 fringes per attracting kilogram. The working conditions below proved to be much better than this, rigidity being estimated too high. If the tenacity of quartz be taken as 1.5×10^3 kg. per square centimeter the latter filament ($r = 10^{-3}$ cm.) should still hold 4.5 grams, or much more than the weight of the above needle (within 2 grams). Having mounted the needle and found the fringes, the image of the wide slit (i.e., the reflected beam seen in the telescope) was apparently quite stationary, the light needle being thus adequately damped. But within this virtually stationary slit image, the fringes (preferably made horizontal) during the winter months continually wandered up and down, showing that micrometric vibration had not been eliminated. The experiment is a very impressive one; but the drift in a heated room is so large that all attempt at measurement is idle.

In the summer installation in a subcellar at nearly constant temperature with a few improvements of apparatus (the mirrors being readjusted, etc.) the needle was without difficulty made to take a stable position midway between the glass plates, subject to the torsion of the fiber. With the micrometer at 45 degrees, the latter is limited in its displacement. I therefore later attached a special micrometer with three identical pairs of parallel V-mirrors (the latter at 90 degrees) as shown at n, n', n'', in figure 4, M, M', N, N', being the interferometer mirrors, L the collimator and T the telescope. This design when the mirrors are reciprocally parallel, has the additional advantage of being independent of slight changes of inclination in the micrometer. The displacement due to V-mirrors is now virtually parallel to the rays and no difficulty in finding the fringes need occur. Naturally the mirrors must be good, there being now four additional reflections in each ray; and the V-apparatus must be accurately adjusted for parallelism of mirrors. The central mirrors, n', only are moved by the micrometer screw.

With the fringes found there is now no difficulty in showing the attraction of gravitation. In fact in iron brick moved on a small truck near the shot at one end of the needle, gripped the balls m, m' very much like a magnet acting on the pole of a magnetic needle. Throughout the whole of the experiment the fringes were under the perfect control of the micrometer.

A more systematic experiment was made by testing the attraction of a lead ball 5.43 cm. in diameter and weighing about $M = 950$ grams, for the shot m' (at the end of the needle) weighing $m' = 0.61$ grams. The kilo was moved on a circular track with stops to a distance of $R = 4.24$ cm. (between centers of balls) from the ball of the needle, alternately. The period of the air-damped needle can not have been less than 18 min. The case is then that of a forced vibration under constant force and a large logarithmic decrement. An example of the observations is given in the graphs figures 5 and 6, the reading being made every minute beginning with the equilibrium position (M in the neutral position). The ordinates of the graphs show the displacement of the attracted shot, m', in cm. The periods of alteration were 10 minutes during the first 110 minutes; thereafter the period of the needle was exceeded. Circles show the time of turning M, from one side of m to the other. The approach to a limiting arc is regular during the first 100 minutes, after which some extraneous disturbance enters; but both for the longer and the shorter intervals of alternation, the tendency is toward the same limiting arc. At the end of the damped period there is a sort of fluttering. The excursions of the needle are so large that it was not thought necessary to read the fringes throughout. In fact, if a micrometer scale is put across the wide slit, there are two methods simultaneously available for finding Δx, the displacement of m due to M. For if f is the focal distance of the collimator and Δy the reading in the telescope of the collimator micrometer scale specified,

$$\Delta x = \cos i \,.\, l\Delta N/b = \Delta y \,.\, l/2f$$

But the reading in terms of ΔN is enormously more accurate than the reading of Δy, the shift of the slit image in the telescope. If the motion of the needle were less nearly dead-beat, the attracting force could be computed from the limiting distance between elongations, if the torsion coefficient of the quartz fiber and the logarithmic decrement were known. From static experiments made during hour intervals this double amplitude was found to be 0.116 cm. or a departure of the shot m at the end of the needle from its position of equilibrium of 0.058 cm. in response to the attraction of M. If l is the semilength of the needle (between centers of shots), the micrometer displacement ΔN and the displacements Δx of the mass m are given by the equation

$$\Delta x = l\Delta N \cos i/b = 0.89 \ \Delta N$$

where b is the breadth of the ray parallelogram and $i = 45$ degrees the angle of incidence of the interferometer. Thus the micrometer displacement is of the same order as the displacement of m, and if the latter is 0.116 cm., we should have

$$\Delta N = 1.3 \text{ cm.}$$

As the micrometer reads to 10^{-4} cm. 1/1300 part of the attraction between M and $m = 0.61$ gram could therefore be detected; i.e., the attraction of 0.73 gram or per interference fringe well within one-third of this, for the given quartz fiber (which was not specially selected) and given distance $R = 4$ cm. This is equivalent to the attraction of 0.2 gram at a distance of 1 cm., per fringe.

Apart from the measurement of the torsion coefficient of the fiber, there is however a real difficulty involved, and that is the occurrence of marked drift in the needle. It is only incidentally that the fringes are found at rest. The chief contributory cause of this is no doubt the occurrence of motion of air around the needle provoked by small differences of temperature, resulting for instance from illumination. If the possible accuracy of deflection measurement is to be of any value, therefore, the apparatus must be kept in the dark except during observation. Fortunately the achromatic fringes require little light. Even then a closed case which can be exhausted of air is essential, for such radiometer forces as may enter would in any event be differential, seeing that the mirrors are symmetric and the illumination is not subject to alternations like the forces on m. It is probable that in case of the above regular method a thicker quartz fiber and a greater distance R would conduce to the best results since ΔN is the least difficult quantity to determine.

The question may now be asked whether it is not possible in the present case to determine gravitational attraction in terms of the acceleration of balls resulting. With an ocular micrometer this would not be difficult as the fringes move slowly enough that the position can always be sharply specified.

But with a needle of long period in vacuum, the screw micrometer would also be available. If there is no damping we may write

$$\gamma Mm/R^2 - ax = 2m\alpha$$

where α is the acceleration, and a the torsion coefficient, referred to x the displacement of m, at the time t, supposing the needle starts from rest, and the gravitational force is applied at $t = 0$. If at the outset we may put $X = \alpha t^2/2$

$$\gamma Mm/R^2 = X(at^2 + 4m)/t^2$$

an equation whose interesting feature is this, that if t is kept very small (which should be possible with an ocular micrometer and the achromatic fringes, a fine quartz fiber presupposed) the term involving t may be neglected and the experiment interpreted as a case of uniformly varied motion in which

$$\gamma = 2R^2\alpha/M$$

Thus for instance if $R = 5$ cm., $M = 10^3$ grams and $\gamma = 6.7 \times 10^{-8}$ and if $t = 100$ seconds is admissible, $\alpha = 1.3 \times 10^{-7}$ cm. /sec.^{2} and the distance traversed in 100 seconds would be 0.0065 cm., well measurable on the interferometer, quite so if the work is done reciprocally and the interference fringes are used individually. The theoretical error will be a minimum if m is as large as the fiber can safely carry and t as small as possible. On the other hand X is independent of m and if t is to be kept small, the result may be compensated in a large M/R^2. The measurement is thus to consist in keeping the fringes at zero by moving the micrometer screw for the small interval t during which the weight M acts. The constant γ would then follow from the micrometer reading, M and R only, all other quantities entering secondarily, as corrections. The experiment seems well worth while.